## CLIL MultiKey lesson plan

## LESSON PLAN

Subject: Mathematics
Topic: Rationalising the denominator.
Age of students: 15-16
Language level: B1-B2
Time: 45 minutes

## General teaching aims:

- To increase students' knowledge of subject content;
- To develop students' ability to interpret and communicate mathematics in a variety of forms;
- To develop all four language skills within content-based context;


## Learning outcomes and objectives:

1. Learners should know:

- The definition of the simplest form of denominators;

2. Learners should be able to:

- Rationalise denominators;
- Use surds properly;

| Content-obligatory language | Content-compatible <br> language |
| :--- | :--- |
| Fraction, numerator, denominator | To get rid of; <br> Square root, surds, <br> Integer, rational number, irrational number <br> To multiply <br> To rationalise the denominator |
| To put something in order <br> Procedure description <br> Result; |  |

## Procedure

1.You know that denominator in the simplest form should not be irrational. You also know from the previous lesson how to change the irrational denominator to rational in examples like :
$\frac{2}{\sqrt{5}} ; \frac{1+\sqrt{2}}{2 \sqrt{2}} ; \ldots \ldots .$.
Today you will learn how to rationalise denominators in different examples.
2.Pair work with worksheet_1 (Problem 1)

Expand the following expressions. Give your answer in the simplest form.
a) $(2+\sqrt{3})(3-\sqrt{3})$
b) $(3-\sqrt{5})(3+\sqrt{5})$
c) $(2 \sqrt{2}-1)(2 \sqrt{2}+1)$
d) $(3 \sqrt{2}+1)^{2}$
e) $(\sqrt{2}+3) \sqrt{2}$
f) $(\sqrt{7}-4)(\sqrt{7}+4)$

When you received rational number as a result ? When you received irrational number as a result ? Can you write an example of expression $(a+b \sqrt{c})(d+e \sqrt{f})$ which give us a rational number as a result? What the rule do you use ?

The rule $(x+y)(x-y)=x^{2}-y^{2}$ is very useful when we want to rationalise denominators in example like :
$\frac{2}{3-\sqrt{5}}$
What should we do ? How can we move square root of 5 to the top ? Think about examples from worksheet_1. Remember that multiplying a fraction by the same thing on the top and bottom will not change the value.

$$
\begin{gathered}
\frac{2}{3-\sqrt{5}}=\frac{2}{3-\sqrt{5}} \cdot \frac{3+\sqrt{5}}{3+\sqrt{5}}=\frac{2(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}=\frac{2(3+\sqrt{5})}{3^{2}-(\sqrt{5})^{2}}=\frac{2(3+\sqrt{5})}{9-5}=\frac{2(3+\sqrt{5})}{4} \\
=\frac{3+\sqrt{5}}{2}
\end{gathered}
$$

3.. Now try to do with others examples. Put the pieces of solution in order.

Students are working in 2-3 persons group. They are working with worksheet_2

| $\frac{3}{2-\sqrt{3}}=$ | $\frac{3}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}}=$ | $\frac{3(2+\sqrt{3})}{2^{2}-(\sqrt{3})^{2}}=$ | $\frac{3(2+\sqrt{3})}{4-3}=$ |
| :---: | :---: | :---: | :---: |
| $\frac{3(2+\sqrt{3})}{1}=$ | $3(2+\sqrt{3})=$ | $6+3 \sqrt{3}$ |  |


| $\frac{2+\sqrt{7}}{1+\sqrt{7}}=$ | $\frac{2+\sqrt{7}}{1+\sqrt{7} \cdot \frac{1-\sqrt{7}}{1-\sqrt{7}}=}$ | $\frac{(2+\sqrt{7})(1-\sqrt{7})}{1^{2}-(\sqrt{7})^{2}}$ | $\frac{2-2 \sqrt{7}+\sqrt{7}-7}{1-7}=$ |
| :---: | :---: | :---: | :---: |
| $\frac{-5-\sqrt{7}}{-6}=$ | $\frac{-(5+\sqrt{7})}{-6}=$ | $\frac{5+\sqrt{7}}{6}$ |  |

Now rationalise the denominator of $\frac{3-\sqrt{3}}{2+\sqrt{3}}$
4.. Express the following in the form $a+b \sqrt{c}$, where $c$ is an integer, and $a$ and $b$ are integers or fraction:
a) $\frac{3 \sqrt{2}}{4-3 \sqrt{2}}$
b) $\frac{2 \sqrt{5}+3}{\sqrt{5}+2}$
c) $\frac{2}{\sqrt{7}-\sqrt{3}}$
5. Put the pieces of procedure description in order (worksheet_2, Problem 4)

| to rationalise the denominator | we multiply top and bottom of the fraction |
| :---: | :---: |
| by the expression | that will get rid of surds in the denominator |

6. Solve the equation $(2 \sqrt{3}+4) x=3 \sqrt{3}-1$ giving your answer in the form $a+b \sqrt{c}$, where $c$ is an integer, and $a$ and $b$ are integers or fraction.
6.Homework: Worksheet_3
7. Express the following in the form $a+b \sqrt{c}$, where $c$ is an integer, and $a$ and $b$ are integers or fraction:
a) $\frac{2 \sqrt{3}+1}{3-2 \sqrt{3}}$
b) $\frac{3 \sqrt{5}}{\sqrt{5}-2}$
c) $\frac{\sqrt{11}+3}{\sqrt{11}-3}$
d) $\frac{2}{\sqrt{5}+\sqrt{3}}$
8. Solve the equation $(\sqrt{13}+4) x=2-\sqrt{13}$ giving your answer in the form $a+b \sqrt{c}$, where $c$ is an integer, and $a$ and $b$ are integers or fraction.

## Worksheet_1

## Problem 1

Expand the following expressions. Give your answer in the simplest form.
a) $(2+\sqrt{3})(3-\sqrt{3})=$
b) $(3-\sqrt{5})(3+\sqrt{5})=$
c) $(2 \sqrt{2}-1)(2 \sqrt{2}+1)=$
d) $(3 \sqrt{2}+1)^{2}=$
e) $(\sqrt{2}+3) \sqrt{2}=$
f) $(\sqrt{7}-4)(\sqrt{7}+4)=$

When you received rational number as a result ?
When you received irrational number as a result ?
Can you write an example of expression $(a+b \sqrt{c})(d+e \sqrt{f})$ which give us a rational number as a result? What the rule do you use ?

## Worksheet_2

## Problem 2

Cut it to 7 pieces before you give it to students.

| $\frac{3}{2-\sqrt{3}}=$ | $\frac{3}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}}=$ | $\frac{3(2+\sqrt{3})}{2^{2}-(\sqrt{3})^{2}}=$ | $\frac{3(2+\sqrt{3})}{4-3}=$ |
| :---: | :---: | :---: | :---: |
| $\frac{3(2+\sqrt{3})}{1}=$ | $3(2+\sqrt{3})=$ | $6+3 \sqrt{3}$ |  |

## Problem 3

Cut it to 7 pieces before you give it to students.

| $\frac{2+\sqrt{7}}{1+\sqrt{7}}=$ | $\frac{2+\sqrt{7}}{1+\sqrt{7} \cdot \frac{1-\sqrt{7}}{1-\sqrt{7}}=}$ | $\frac{(2+\sqrt{7})(1-\sqrt{7})}{1^{2}-(\sqrt{7})^{2}}=$ | $\frac{2-2 \sqrt{7}+\sqrt{7}-7}{1-7}=$ |
| :---: | :---: | :---: | :---: |
| $\frac{-5-\sqrt{7}}{-6}=$ | $\frac{-(5+\sqrt{7})}{-6}=$ | $\frac{5+\sqrt{7}}{6}$ |  |

Problem 4
Cut it to 4 pieces before you give it to students.

| to rationalise the denominator | we multiply top and bottom of the fraction |
| :---: | :--- |
| by the expression | that will get rid of surds in the denominator |

## Worksheet_3

## Homework

1. Express the following in the form $a+b \sqrt{c}$, where $c$ is an integer, and $a$ and $b$ are integers or fraction:
a) $\frac{2 \sqrt{3}+1}{3-2 \sqrt{3}}=$
b) $\frac{3 \sqrt{5}}{\sqrt{5}-2}=$
c) $\frac{\sqrt{11}+3}{\sqrt{11}-3}=$
d) $\frac{2}{\sqrt{5}+\sqrt{3}}=$
2. Solve the equation $(\sqrt{13}+4) x=2-\sqrt{13}$ giving your answer in the form $a+b \sqrt{c}$, where $c$ is an integer, and $a$ and $b$ are integers or fraction.
