LESSON PLAN

Subject: Mathematics

Topic: Logarithms

Age of students: 16-17

Language level: B1-B2

Time: 90-120 minutes

Content aims:

- To increase students' knowledge of subject content;
- To develop students' ability to interpret and communicate mathematics in a variety of forms;

Language aims:

- To apply new vocabulary in the frames of the topic;
- To interpret and communicate mathematics;
- To develop all four language skills within content-based context;

Pre-requisites:

Index notation

Laws of powers

Key words: power,index notation,log notation,natural logarithms, common logarithm, logarithm of b with base a.

Assessment:

Class observations, questions. Teacher will also collect information through informal assessment activities. By observing students as they work in groups, in pairs, asking questions and marking students' work.

Procedure

Introduction (real-word connection)

As you know A tornado is a tube of violently spinning air that touches the ground. Wind inside the tornado spins fast, but the actual 'circle' of wind around them is massive.

The wind speed s (in miles per hour) near the centre of the tornado can be modeled by: $s = 93 \log d + 65$; where d is the distance (in miles) that the tornado travels. This formula uses logarithm, which you will study in this lesson.

Exercise 1. [individual work]

Find the value of:

a) 4^2 b) $25^{\frac{1}{2}}$ c) 2^{-3}

A logarithm is just the power that a number needs to be **raised to** produce a given value.

Before now, you have used index notation to represent powers, but sometimes it's easier to work with log notation.

Log notation look like this:

$$\log_a b = c$$

Which means the same as the index notation:

$$a^c = b$$

Therefore:

a) $log_4 16 = 2$ b) $\log_{25} 5 = \frac{1}{2}$ c) $\log_2 \frac{1}{8} = -3$

Logarithm of 16 to the base 4 is 2, because 4 raised to the power of 2 is 16

Logarithm of 5 to the base 25 is $\frac{1}{2}$, because 25 raised to the power of $\frac{1}{2}$ is 5.

Logarithm of $\frac{1}{8}$ to the base 2 is -3, because 2 raised to the power of -3 is $\frac{1}{8}$

Exercise 2. *[Work in pairs – each students has his own sheet and after solving 5 problems he/she compares result with person in pair. The examples for both students are the same, but in different notation.]*

Student 1: Write the following using log notation:

$a)\ 2^5 = 32 \qquad b)\ 6^{-2} = \frac{1}{36} \qquad c)\ 125^{\frac{1}{3}} = 5 \qquad d)\ 7^0 = 1 \quad e)\ 12^1 = 12$

Student 2: Write the following using index notation:

1) $log_7\ 1 = 0$ 2) $log_{12}\ 12 = 1$ 3) $log_2\ 32 = 5$ 4) $log_6\ \frac{1}{36} = -2$
5) $log_{125}\ 5 = \frac{1}{3}$

[After discussing Example 2 with all students together I will stress that the base is the same in both forms just written in a different position, and that the logarithm is the exponent]

$\log_a b = c \iff a^c = b$

- The little number *'a'* after *'log'* is **the base**
- *'c'* is the power the base is being **raised to**
- *'b'* is the answer you get when ***a*** is raised to the power **c**.

Log means **'power'**, so the log above really just means: **what is the power you need to raise *a* to if you want to end up with *b*.**

Example 3:

Write down the value of $log_3\ 9$

- Compare to $log_a b = c$. Here the base *(a)* is 3. And the answer *(b)* is 9.
- So think about the power *(c)* that you will need to raise 3 to to get 9.
- 9 is 3 raised to power of 2, so $3^2 = 9$ and $log_3\ 9 = 2$

Example 4:

Write down the value of $log_{16}\ 4$

- Work out the power that 16 needs to be raised to to get 4.
- 4 is the square root of 16, or $16^{\frac{1}{2}} = 4$, so $log_{16}\ 4 = \frac{1}{2}$.

Exercise 5. [I will divide the class into teams of five students. Each team will choose one problem and has two minutes to solve it. In case of wrong answer another team has chance to achieve a point]

Find the value of:

$a)\ \log_2 2 =$ $b)\ \log_{\frac{1}{3}} 9 =$ $c)\ \log_5 0{,}2 =$ $d)\ \log_{25} 5 =$

$e)\ \log_{0,5} 2 =$ $f)\ \log_7 1 =$ $g)\ \log_8 \frac{1}{3} =$ $h)\ \log_{\frac{1}{4}} 4 =$

$h)\ \log_{\frac{1}{5}} 25 =$ $i)\ \log_{\frac{1}{7}} 7 =$

Remember:

- The base of a log must always be a positive number except 1.
- You can't take a log of a negative numbers (there's no power you can raise a positive number to to make it negative)

You can get logs to any base, but base 10 is the most common. The base is left out if it's 10. The button marked *'log'* on your calculator uses base 10. (of course some calculators allow you to find logs to any base).

Exercise 6. [individual work]

Find the value of:

$a)\ log100$ $b)\ log\ 0{,}001$ $c)\ log\sqrt{10}$ $d)\ log\ 1$

Exercise 7. [*Work in pairs- both students have the same problems and they work together]*

Using your calculator find value of common logarithms and put your results in right order.

$log35$; $log9$; $log\ 0{,}234$; $log\ 1205$; $log101$; $log0{,}002$

As $\log_a b = c$ is the same as $a^c = b$, it means that:

$$\log_a a = 1 \quad \text{and} \quad \log_a 1 = 0$$

Exercise 8. Find the value of x, where $x > 0$, by writing the following in index notation:

$a)\ \log_{0,5} x = 2$ $\quad b)\ \log_x \frac{1}{81} = 4$ $\quad c)\ \log_6 x = \frac{1}{2}$ $\quad d)\ \log_x 64 = 3$

Now we can back to formula connected with tornado. The wind speed s (in miles per hour) near the centre of the tornado can be modeled by: $s = 93 \log d + 65$; where d is the distance (in miles) that the tornado travels.

In 1925 a tornado traveled 220 miles through three states of USA. Estimate the wind speed near the tornado's centre.

[work in pairs – the student from the first pair will show solution on the board]

Answer: The wind speed near the tornado's centre was about 283 miles per hour.

Homework:

Exercise 1. Write the following using log notation:

$a)\ 7^2 = 49$ $\quad b)\ 3^{-3} = \frac{1}{27}$ $\quad c)\ 6^{\frac{1}{2}} = \sqrt{6}$ $\quad d)\ 10^4 = 10000$

Exercise 2. Find in the net the definition of natural logarithm and evaluate natural logarithms using your calculator :

a) $ln4 =$ b) $ln12$ = c) $ln2 =$ d) $ln0.5 =$

Exercise 3. Find the value of x, where $x > 0$, by writing the following in index notation:

$a)\ \log_{0,1} x = 2$ $\quad b)\ \log_x \frac{1}{27} = 3$ $\quad c)\ \log_4 x = \frac{1}{2}$ $\quad d)\ \log_x 8 = 3$

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Exercise 4. Find the path so that each arrow leads to a correct answer.

START $\log_2 8$	-2 $\Rightarrow$	$\log_5 5$	+1 $\Rightarrow$	$\log_3 27$	+1 $\Rightarrow$	$\log_2 \frac{1}{8}$
$\Downarrow$ *3		$\Downarrow$ *5		$\Downarrow$ *5		$\Downarrow$
$\log 100$	-2 $\Rightarrow$	$\log_2 32$	-2 $\Rightarrow$		-2 $\Rightarrow$	$\log 0.01$
$\Downarrow$ -3		$\Downarrow$ -3		$\Downarrow$ *5		$\Downarrow$ +3
$\log_5 1$	-2 $\Rightarrow$	$\log_{0.5} 0.25$	-2 $\Rightarrow$		-2 $\Rightarrow$	
$\Downarrow$		$\Downarrow$		$\Downarrow$ *5		$\Downarrow$ *5
$\log_{\sqrt[3]{3}} 3$	-2 $\Rightarrow$	$\log_{\sqrt{2}} 2$	-4 $\Rightarrow$	$\log_4 \frac{1}{16}$	+2 $\Rightarrow$	$\log 1$ THE END