LESSON PLAN

Subject: Mathematics

Topic: Area of triangle, Heron´s formula

Content aims:

After completing the lesson, the student will be able to:

Describe Pythagorean theorem.

Explain Heron´s formula.

Derivate Heron´s formula.

Perform mathematical operations and manipulations with Heron's formula.

Language aims:

After completing the lesson, the student will be able to:

Use new vocabulary within the topic.

Interpret and communicate mathematics.

Procedure:

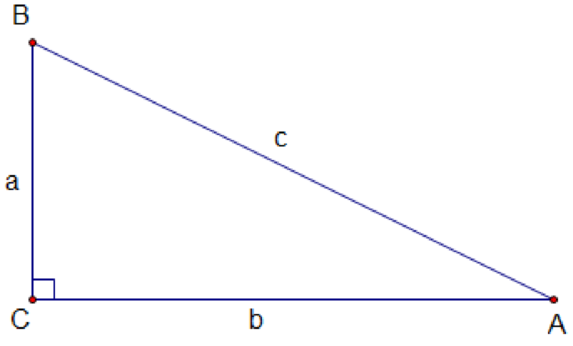
1. Repeating formulas for area of triangle:

a) Right angle triangle:

Area, right angle triangle, hypothenuse

To calculate

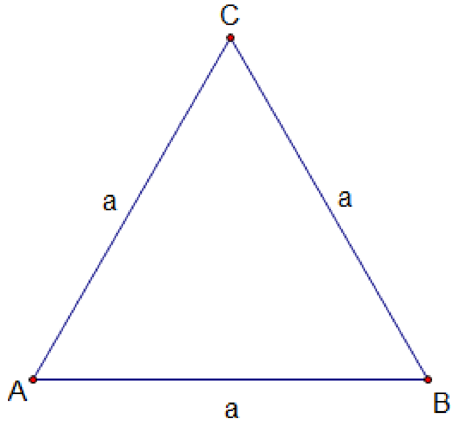
$$A = \frac{a \cdot b}{2}$$



Question:

Question 1: Find the length of the hypotenuse of the right triangle if the length of the other two sides are 5 cm and 7 cm. Also calculate the area of the triangle. *Using the Pythagorean theorem.*

b) Equilateral triangle

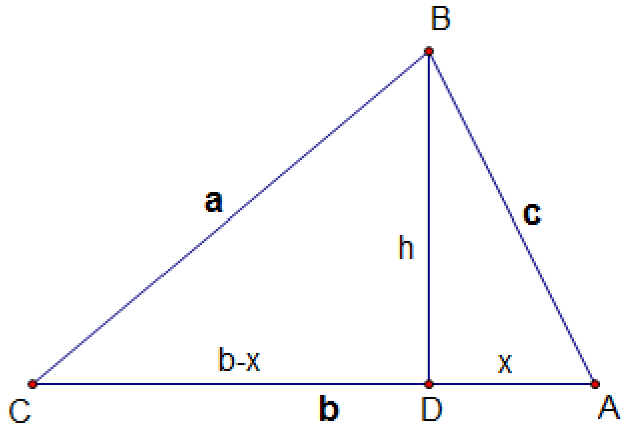


$$A = \frac{a^2\sqrt{3}}{4}$$

2. How to find the area of a triangle if 3 sides are given?

There is an additional formula, called Heron's formula, that will let you determine the area of a triangle given the lengths of the three sides.

Deriation of Heron´s formula for triangle ABC:



Uquilateral triangle

To determine, to derivate

Area of triangle ABC:

$$A = \frac{1}{2}bh \qquad \text{-equation (1)}$$

From triangle CBD:

$$(b-x)^2 + h^2 = c^2$$

$$x^2 = c^2 - h^2$$

$x = \sqrt{c^2 - h^2}$

From triangle CBD:

$$(b-x)^2 + h^2 = a^2$$
$$(b-x)^2 = a^2 - h^2$$
$$b^2 - 2bx + x^2 = a^2 - h^2$$

Perimeter, semiperimeter, equation

To substitute, to derivate

Substitute the values of x and x^2

$$b^2 - 2b\sqrt{c^2 - h^2} + \left(c^2 - h^2\right) = a^2 - h^2$$
$$b^2 + c^2 - a^2 = 2b\sqrt{c^2 - h^2}$$

Square both sides

$$\left(b^2 + c^2 - a^2\right)^2 = 4b^2\left(c^2 - h^2\right)$$
$$\frac{\left(b^2 + c^2 - a^2\right)^2}{4b^2} = c^2 - h^2$$
$$h^2 = c^2 - \frac{\left(b^2 + c^2 - a^2\right)^2}{4b^2}$$
$$h^2 = \frac{4b^2c^2 - \left(b^2 + c^2 - a^2\right)^2}{4b^2}$$
$$h^2 = \frac{(2bc)^2 - \left(b^2 + c^2 - a^2\right)^2}{4b^2}$$
$$h^2 = \frac{\left(2bc + b^2 + c^2 - a^2\right)\left(2bc - b^2 - c^2 + a^2\right)}{4b^2}$$
$$h^2 = \frac{\left[\left(b^2 + 2bc + c^2\right) - a^2\right]\left[a^2 - \left(b^2 - 2bc + c^2\right)\right]}{4b^2}$$
$$h^2 = \frac{(a+b+c)(b+c-a)(a+c-b)(a+b-c)}{4b^2}$$

$$h^2 = \frac{(a+b+c)(a+b+c-2a)(a+b+c-2b)(a+b+c-2c)}{4b^2}$$

P = perimeter, $P = a+b+c$

$$h^2 = \frac{P(P-2a)(P-2b)(P-2c)}{4b^2}$$

$$h = \frac{\sqrt{P(P-2a)(P-2b)(P-2c)}}{2b}$$

Substitute h to equation (1)

$$A = \frac{1}{2} b \frac{\sqrt{P(P-2a)(P-2b)(P-2c)}}{2b}$$

$$A = \frac{1}{4}\sqrt{P(P-2a)(P-2b)(P-2c)}$$

$$A = \sqrt{\frac{1}{16} P(P-2a)(P-2b)(P-2c)}$$

$$A = \sqrt{\frac{P}{2}\left(\frac{P}{2}-a\right)\left(\frac{P}{2}-b\right)\left(\frac{P}{2}-c\right)}$$

Recall that $\frac{P}{2} = s$ - Semiperimeter

$$A = \sqrt{s(s-a)(s-b)(s-c)}\ .$$

Practise:

Use Heron's formula to find the area of the triangle with the following side lengths and fill the table:

a) $a = 30$ cm, $b = 25$ cm, $c = 11$ cm;

b) $a = 70$ cm, $b = 65$ cm, $c = 9$ cm;

c) $a = 80$ cm, $b = 73$ cm, $c = 9$ cm;

d) $a = 37$ cm, $b = 20$ cm, $c = 19$ cm;

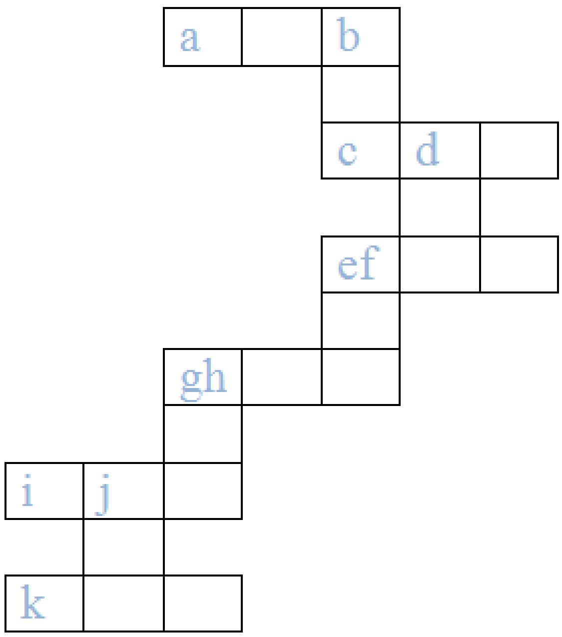
e) $a = 40$ cm, $b = 37$ cm, $c = 13$ cm;

f) $a = 52$ cm, $b = 41$ cm, $c = 15$ cm;

g) $a = 26$ cm, $b = 25$ cm, $c = 17$ cm;

h) $a = 29$ cm, $b = 21$ cm, $c = 20$ cm;

i) $a = 28$ cm, $b = 25$ cm, $c = 17$ cm;



j) $a = 52$ cm, $b = 51$ cm, $c = 5$ cm;

k) $a = 48$ cm, $b = 25$ cm, $c = 25$ cm;

Solution:

		1	3	2		
				5		
				2	1	6
					1	
				2	4	0
				3		
		1	1	4		
		2				
2	1	0				
	2					
1	6	8				