MATH AND ART WALK TOGETHER

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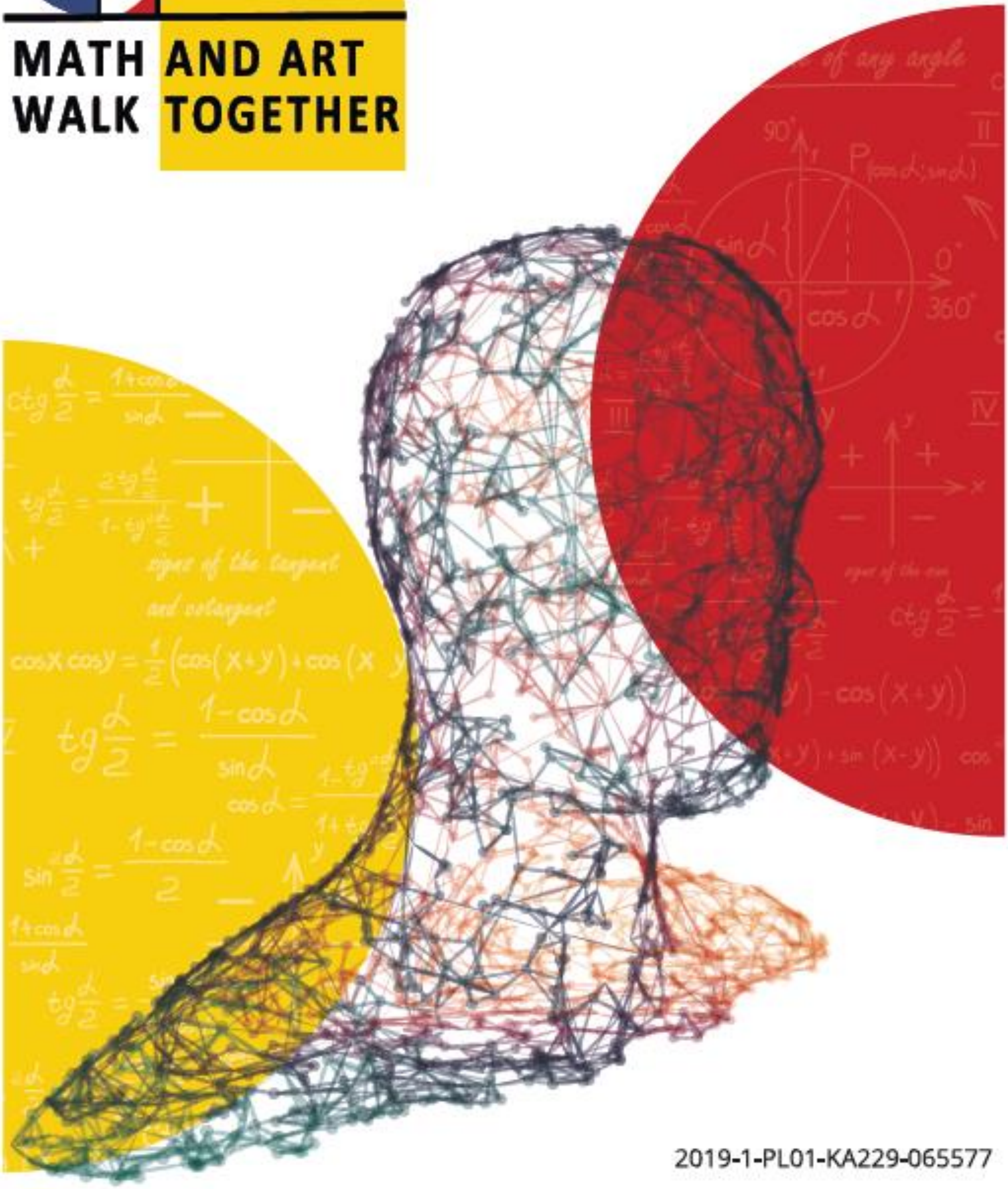

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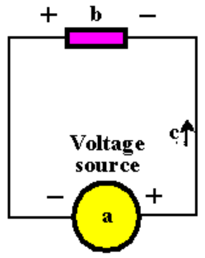
Math and Art Walk Together in Electronic

CONTENTS:

Some definitions and formulas for tasks’

Ohm's law: the voltage across a resistor is directly proportional to the current flowing through the resistance at constant temperature.

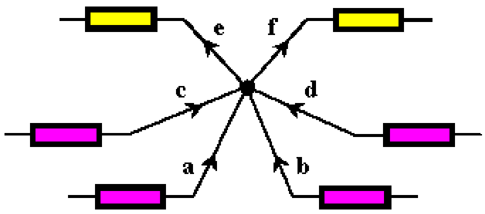


Formula 1: $E_a = R_b \times I_c$

Electric power: the rate at which energy gets transported to or from a part of an electric circuit.

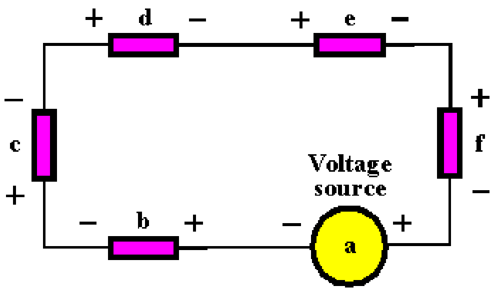
Formula 2: $P = U \times I$

Kirchhoff’s first law (Current law): the algebraic sum of currents leaving the node in a network is equal to the algebraic sum of currents entering the same node.



Formula 3: $I_a + I_b + I_c + I_d = I_e + I_f$

Kirchhoff’s second law (Voltage law): the net electromotive force around a closed circuit loop is equal to the sum of potential drops around the loop (formula 4). Or, the algebraic sum of voltages dropped across components around a loop is zero (formula 5).



Formula 4: $E_a = U_b + U_c + U_d + U_e + U_f$

Formula 5: $0 = -E_a + U_b + U_c + U_d + U_e + U_f$

Simple practical tasks in electrical engineering'

Task 1:

Find out the value of resistance and the power dissipated by the resistor in the circuit (fig. 1). Known quantities: $E = 3.3V$, $U_D = 1.9V$, $I = 10mA$.

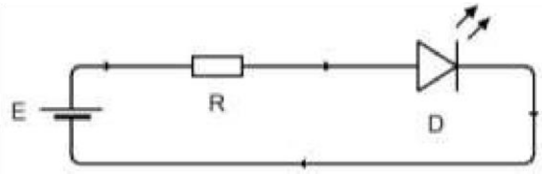


fig. 1

Task 2:

Find out the value of current drawn from a supply in the circuit (fig. 2). What happens to the voltage across each branch of the circuit? Known quantities: $E=3.3V$, $U_D=1.9V$, $R=140\Omega$, $I_R=10mA$.

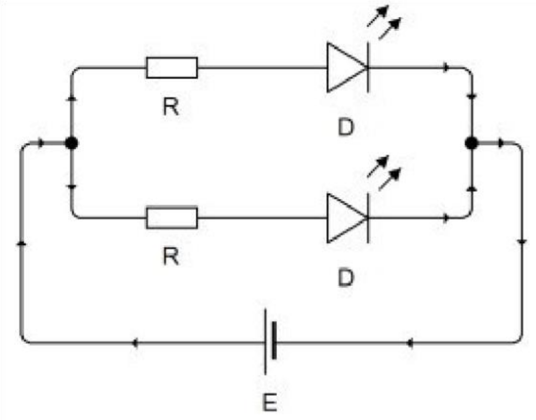


fig. 2

Task 3A:

Check if the current source is enough to supply the circuit (fig. 3). Known quantities: $E=3.3V$, $U_D=1.9V$, $R=140\Omega$, $I=10mA$

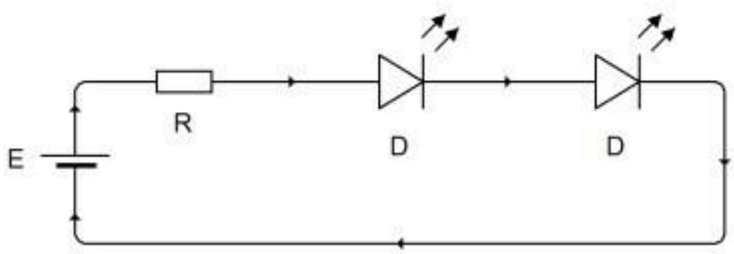


fig. 3

Task 3B:

Find the value of the current in the circuit and the voltage dropped across the resistor (fig. 3). Known quantities: $E=3.3V$, $U_D=1.4V$, $R=140\Omega$

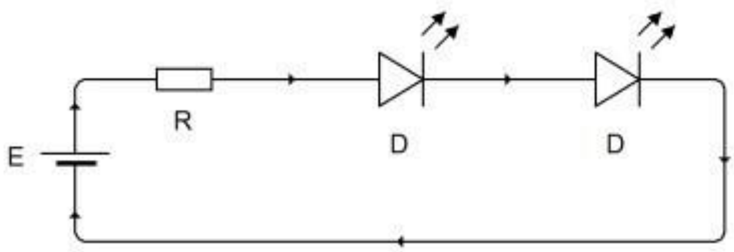


fig. 3

Practical assignment ‘traffic lights’

Do the calculations for the diagram in Figure 1, then answer the questions and fill in the table with the results of your calculations. Select the nominal value of the resistor from the main series table.

Data:

Voltage source – (pin GPIO26, pin GPIO19, pin GPIO13):

- ε: (voltage on the pins) = 3.3 [V],
- P_{max} GPIO : (maximum output power) = 52.8 [mW]

D1 – (LED red colour):

- U_{D1} : (conduction voltage U_f) = 1.6 [V], (maximum voltage U_f max) = 2.2 [V],
- I_{D2} : (maximum conduction current I_f max) = 20 [mA].

D2 – (LED yellow colour):

- U_{D2} : (conduction voltage U_f) = 2.0 [V], (maximum voltage U_f max) = 2.3 [V],
- I_{D2} : (maximum conduction current I_f max) = 20 [mA].

D3 – (LED green colour):

- U_{D3} : (conduction voltage U_f) = 2.0 [V], (maximum voltage U_f max) = 3.7 [V],
- I_{D3} : (maximum conduction current I_f max) = 20 [mA].

Required calculations:

- the electrical resistance for resistors R1, R2, R3 ,
- real resistance for resistors R1, R2, R3 (depending on the selected series),
- voltage drop on resistors (U_{R1}, U_{R2}, U_{R3}) including drop on real resistors,
- the output power (P_{R1}, P_{R2}, P_{R3}) generated from resistors including the dropping of real resistors,
- energy (E_{R1}, E_{R2}, E_{R3}) emitted at resistors at time t=1 week including real resistors.

Questions:

- Does the maximum power rating of a single resistor was exceeded ($P_{max} \leq 250$ [mA])?
- Can the variation in tolerance of the selected real resistors (R1, R2, R3) damage the circuit in Figure 1?
- What is the total power of the system in Figure 1 (what minimum power source is required for the system)?
- What is the total energy of the system in Figure 1 (what will be the current consumption at time t)?

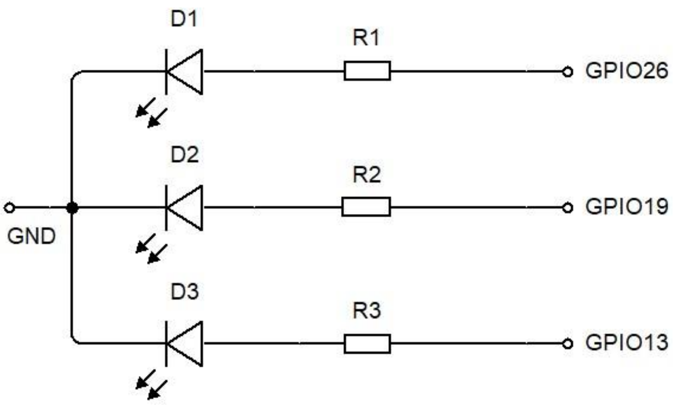


Figure 1: Electronic circuit diagram.

Table for summary of calculation results

Calculation results table:

	R1				R2				R3			
		Series:				Series:				Series:		
tolerance	Calculated value	selected	% series tolerance	% real rate	Calculated value	selected	% series tolerance	% real rate	Calculated value	selected	% series tolerance	% real rate
R[Ω] – resistance												
I_D[A] – current flowing though the diode (half value I_f max)												
U_R[V] – voltage drop at the resistor												
P_R[W] – power dissipated by a resistor												
E_R[Wh] – energy dissipated by a resistor at time t												
U_D[V] – voltage drop in diode (central value)												
P_D[W]– power dissipated in diode												
E_D[Wh] – energy dissipated by diode at time t												

tab. 1: Summary of calculation results.

Solution to the simple practical tasks in electrical engineering

MATH AND ART WALK TOGETHER

Task 1:

Find out the value of resistance and the power dissipated by the resistor in the circuit (fig. 1). Known quantities: $E = 3.3V$, $U_D = 1.9V$, $I = 10mA$.

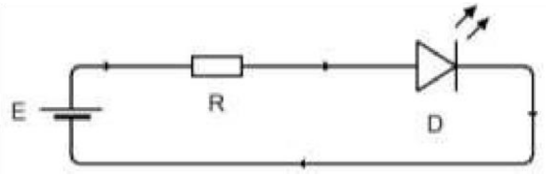


fig. 1

Solution:

Step 1: Using Kirchhoff's second law.

$$E = U_R + U_D$$

Where:
E - electromotive force,
U_R – drop of potential on a resistor,
U_D – drop of potential on a light emitting diode (LED)

Step 2: Substitute known quantities into equation from step 1.

$$3.3V = U_R + 1.9V$$

This will give:

$$U_R = 3.3V - 1.9V = 1.4V$$

Step 3: Using Ohm's law.

$$U_R = R \times I$$

Where:
R – value of resistance at a resistor,
I – current flowing through a resistor

Step 4: Substitute known and calculated quantities into equation from step 3.

$$1.4V = R \times 10mA$$

This will give:

$$R = 1.4V/10mA = 1.4V/0.01A = \underline{\mathbf{140\Omega}}$$

Step 5: Using the electric power formula.

$$P = U_R \times I$$

Where:
P – the electric power dissipated by a resistor,
U_R – drop of potential on a resistor,
I – current flowing through a resistor

Step 6: Substitute known and calculated quantities into equation from step 5.

$$P = 1.4V \times 10mA$$

This will give:

$$P = 1.4V \times 0.01A = \mathbf{0.014W} = \mathbf{14mW}$$

Answer: the value of resistance in the task circuit is **<u>140Ω</u>**. The electric power dissipated by the resistor is **0.014W**, which is equivalent to **14mW**.

Task 2:

Find out the value of current drawn from a supply in the circuit (fig. 2). What happens to the voltage across each branch of the circuit? Known quantities: $E=3.3V$, $U_D=1.9V$, $R=140\Omega$, $I_R=10mA$.

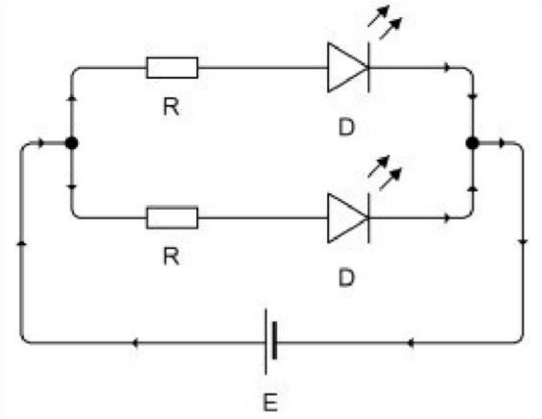


fig. 2

Solution:

Step 1: Using Kirchhoff's current law.

$$I = I_R + I_R = 2I_R$$

Where:
I – current drawn from a supply,
I_R – current flowing through a resistor,

This will give:

$$I = 2 \times 10mA = \underline{\mathbf{20mA}}$$

Answer: The current of the circuit in a parallel connection is the sum of the currents of each node up to maximum current output of a power supply. The current drawn from a supply for the circuit in the task is 20mA.

Step 2: Using Ohm's law and Kirchhoff's voltage law.

$$\text{Ohm's law: } U_R = R \times I$$
$$\text{KVL: } E = U_R + U_D$$

Where:
E - electromotive force,
U_R – drop of potential on a resistor,
U_D – drop of potential on a light emitting diode (LED),
R – value of resistance at a resistor

This will give:

$$U_R = R \times I = 140\Omega \times 10mA = 140\Omega \times 0.01A = 1.4V$$
$$E = 1.9V + 1.4V = 3.3V$$

Answer: According to the Kirchhoff's voltage law, the potential drop at each node is the same as the current source.

Task 3A:
Check if the current source is enough to supply the circuit (fig. 3). Known quantities: $E=3.3V$, $U_D=1.9V$, $R=140\Omega$, $I=10mA$

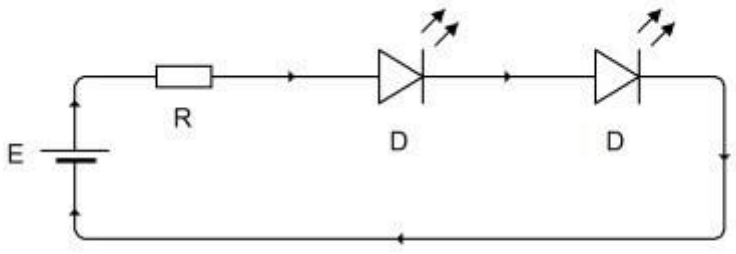


fig. 3

Solution A:

Step 1: Using Ohm's law.

$$U_R = R \times I$$

Where:
U_R – drop of potential on a resistor,
R – value of resistance at a resistor,
I – current flowing through components in the circuit.

This will give:

$$U_R = 140\Omega \times 10mA = 140\Omega \times 0.01A = 1.4V$$

Step 2: Using Kirchhoff's voltage law.

$$E = U_R+U_D$$

Where:
E - electromotive force.

This will give:

$$E = 1.4V + 2 \times 1.9V = 4.2V$$
$$3.3V \neq 5.2V$$
$$5.2V – 3.3V = 1.9V$$

Answer: The power source does not offer sufficient voltage to supply the circuit components. The voltage of the power supply is too low about 1.9V, which correlates to one diode from the task.

Task 3B:

Find the value of the current in the circuit and the voltage dropped across the resistor (fig. 3). Known quantities: $E=3.3V$, $U_D=1.4V$, $R=140\Omega$

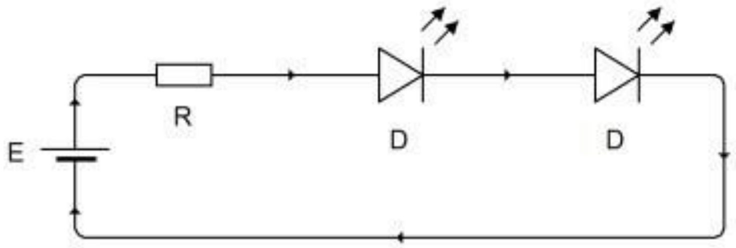


fig. 3

Solution B:

Step 1: Using Kirchhoff's voltage law.

$$E = U_R+2 \times U_D$$

Where:
E - electromotive force,
U_R – drop of potential on a resistor,
U_D – drop of potential on a light emitting diode (LED).

This will give:

$$U_R = E - 2 \times U_D = 3.3V - 2 \times 1.4V = \underline{\mathbf{0.5V}}$$

Step 2: Using Ohm's law.

$$U_R = R \times I$$

Where:
R – value of resistance at a resistor
I – current flowing through components in the circuit.

This will give:

$$I = U_R / R = 0.5V / 140\Omega = \underline{\mathbf{3.57mA}}$$

Answer: an approximately value of the current in the circuit is 3.57 mA. Drop of the potential across a resistor reaches 0.5V, which is a nominal value of 500mV.

Results of measurements and calculations for the practical assignment

Calculation results table.

The results entered in the table are in blue:

tolerance	R1				R2				R3			
	Calculated value	Series: E24- step 4			Calculated value	Series: E24- step 4			Calculated value	Series: E24- step 4		
		selected	5% - series tolerance	1.682% - real rate		selected	5% - series tolerance	2.182% - real rate		selected	5% - series tolerance	1.4% - real rate
R[Ω] – resistance – step 3	140[Ω]	220[Ω]	209[Ω] 231[Ω]	216.3[Ω]	115[Ω]	220[Ω]	209[Ω] 231[Ω]	215.2[Ω]	45[Ω]	100[Ω]	95[Ω] 105[Ω]	98.6[Ω]
I_D[A] – current flowing though the diode (half value I_f max) – step 1, step 5	10[mA]			5.52[mA]	10[mA]			5.62[mA]	10[mA]			8.68[mA]
U_R[V] – voltage drop at the resistor – step 5	1.4[V]			1.27[V]	1.15[V]			1.26[V]	0.45[V]			1.02[V]
P_R[W] – power dissipated by a resistor – step 6	14[mW]			7.01[mW]	11.5[mW]			7.08[mW]	4.5[mW]			8.85[mW]
E_R[Wh] – energy dissipated by a resistor at time t – step 7	2.35[Wh]			1.18[Wh]	1.93[Wh]			1.19[Wh]	0.76[Wh]			1.49[Wh]
U_D[V] – voltage drop in diode (central value) – step 2, step 5	1.9[V]			1.84[V]	2.15[V]			1.85[V]	2.85[V]			2.09[V]
P_D[W]– power dissipated in diode – step 6	19[mW]			10.49[mW]	21.5[mW]			12.08[mW]	28.5[mW]			24.74[mW]
E_D[Wh] – energy dissipated by diode at time t – step 7	3.19[Wh]			1.76[Wh]	3.61[Wh]			2.03[Wh]	4.79[Wh]			4.16[Wh]

tab. 1: Summary of calculation results.

Step 1: Half of the maximum current flow through the diodes was used in the calculations, that is 10[mA].

Step 2: Calculation of the voltage drop across diodes D1, D2, D3. The value of the voltage drop across the diodes is the central value between the minimum and maximum conduction values. Used formula for calculations: $U = (U_f max - U_f)/2 + U_f$

D1 – U_{D1} : (conduction voltage U_f) = 1.6[V], (maximum voltage U_f max) = 2.2[V]

U_{D1} = (2.2[V] – 1.6[V])/2 + 1.6[V] = 1.9[V]

D2 – U_{D2} : (conduction voltage U_f) = 2.0[V], (maximum voltage U_f max) = 2.3[V]

U_{D2} = (2.3[V] – 2.0[V])/2 + 2.0[V] = 2.15[V]

D3 – U_{D3} : (conduction voltage U_f) = 2.0[V], (maximum voltage U_f max) = 3.7[V]

U_{D3} = (3.7[V] – 2.0[V])/2 + 2.0[V] = 2.85[V]

Step 3: The electrical resistance of resistors R1, R2, R3 was calculated using Ohm's law and Kirchhoff's voltage law.
Used formulas for calculations: $\varepsilon = U_d + U_R$, $U_R = R \times I$. The result is this formula: $R= (\varepsilon - U_d)/I$

R1 = (3.3[V] - 1.9[V])/10[mA] = 1.4[V] × 100[A] = 140[Ω]

R2 = (3.3[V] - 2.15[V])/10[mA] = 1.15[V] × 100[A] = 115[Ω]

R3 = (3.3[V] - 2.85[V])/10[mA] = 0.45[V] × 100[A] = 45[Ω]

Step 4: The real resistance for resistors R1, R2, R3 (physically available resistors from the E24 series).

R1 = 220[Ω], R2 = 220[Ω], R3 = 100[Ω]

Tolerance in ohms for a selected resistor: R1 = R2 = 220[Ω] × 5% = 11[Ω], R3 = 100[Ω] × 5% = 5[Ω]

Tolerance range: R1 = R2 = (209[Ω] - 231[Ω]), R3 = (95[Ω] - 105[Ω])

The actual (real) resistances for R1, R2, R3 were measured with a digital multimeter: R1 = 216.3[Ω], R2 = 215.2[Ω], R3 = 98.6[Ω]

Difference from assumed resistance value in percentage: R1 = (220[Ω] - 216.3[Ω])/ 220[Ω] ≈ 1.682%, R2 ≈ 2.182%, R3 ≈ 1.4%

Conclusions: Measurements and calculations show that the selected resistors are within the manufacturer's tolerance.

Step 5: Voltage drops across resistors R1, R2, R3 determined from the calculations in step 2. Kirchhoff's voltage law was used in the calculations.

$U_{R1} = \varepsilon - U_{D1} = 3.3[V] - 1.9[V] = 1.4[V]$

$U_{R2} = \varepsilon - U_{D2} = 3.3[V] - 2.15[V] = 1.15[V]$

$U_{R3} = \varepsilon - U_{D3} = 3.3[V] - 2.85[V] = 0.45[V]$

Real (measured) current flowing through each diode was measured with an electronic multimeter:

$I_{D1} = 5.52[mA]$, $I_{D2} = 5.62[mA]$, $I_{D3} = 8.68[mA]$.

Real (measured) voltage drop across resistors and diodes were measured with an electronic multimeter:

$U_{R1} = 1.27[V]$, $U_{R2} = 1.26[V]$, $U_{R3} = 1.02[V]$

$U_{D1} = 1.84[V]$, $U_{D2} = 1.85[V]$, $U_{D3} = 2.09[V]$

Conclusions: Following Kirchhoff's voltage law, the source voltage is equal to the sum of the voltage drops across the components in the circuit. In fact, the measured voltage value is 3.297[V] and is the same on all three voltage sources. The missing value of the voltage drop is due to the internal resistance of the voltage source.

$\varepsilon = U_R + U_D + R_W \times I \Rightarrow R_W = (\varepsilon - U_R - U_D) / I$

$R_{W1} = (\varepsilon - U_{R1} - U_{D1}) / I_1 = (3.297[V] - 1.27[V] - 1.84[V]) / 5.52[mA] = 0.187[V] / 5.52[mA] = 33.88[\Omega]$

$R_{W2} = (\varepsilon - U_{R2} - U_{D2}) / I_2 = (3.297[V] - 1.26[V] - 1.85[V]) / 5.62[mA] = 0.187[V] / 5.62[mA] = 33.27[\Omega]$

$R_{W3} = (\varepsilon - U_{R3} - U_{D3}) / I_3 = (3.297[V] - 1.02[V] - 2.09[V] / 8.68[mA] = 0.187[V] / 9.68[mA] = 21.54[\Omega]$

Step 6: The output power generated from resistors (P_{R1}, P_{R2}, P_{R3}) and diodes (P_{D1}, P_{D2}, P_{D3}) is calculated from the power formula $P = I \times U$.

$P_{R1} = I \times U_{R1} = 10[mA] \times 1.4[V] = 0.01[A] \times 1.4[V] = 0.014[W] = 14[mW]$

$P_{R2} = I \times U_{R2} = 10[mA] \times 1.15[V] = 0.01[A] \times 1.15[V] = 0.0115[W] = 11.5[mW]$

$P_{R3} = I \times U_{R3} = 10[mA] \times 0.45[V] = 0.01[A] \times 0.45[V] = 0.0045[W] = 4.5[mW]$

$P_{D1} = I \times U_{D1} = 10[mA] \times 1.9[V] = 0.01[A] \times 1.9[V] = 0.019[W] = 19[mW]$

$P_{D2} = I \times U_{D2} = 10[mA] \times 2.15[V] = 0.01[A] \times 2.15[V] = 0.0215[W] = 21.5[mW]$

$P_{D3} = I \times U_{D3} = 10[mA] \times 2.85[V] = 0.01[A] \times 2.85[V] = 0.0285[W] = 28.5[mW]$

Real power dissipated from resistors.

$P_{R1} = I_{D1} \times U_{R1} = 5.52[mA] \times 1.27[V] = 0.00552[A] \times 1.27[V] = 0.0070104[W] = 7.01[mW]$

$P_{R2} = I_{D2} \times U_{R2} = 5.62[mA] \times 1.26[V] = 0.00562[A] \times 1.26[V] = 0.0070812[W] = 7.08[mW]$

$P_{R3} = I_{D3} \times U_{R3} = 8.68[mA] \times 1.02[V] = 0.00868[A] \times 1.02[V] = 0.0088536[W] = 8.85[mW]$

Real power dissipated from diodes:

$P_{D1} = I_{D1} \times U_{D1} = 5.52[mA] \times 1.9[V] = 0.00552[A] \times 1.9[V] = 0.010488[W] = 10.49[mW]$

$P_{D2} = I_{D2} \times U_{D2} = 5.62[mA] \times 2.15[V] = 0.00562[A] \times 2.15[V] = 0.012083[W] = 12.08[mW]$

$P_{D3} = I_{D3} \times U_{D3} = 8.68[mA] \times 2.85[V] = 0.00868[A] \times 2.85[V] = 0.024738[W] = 24.74[mW]$

Step 7: Energy emitted at resistors and diodes at time t=1 week = 168 hours.

$E_{R1} = P_{R1} \times t = 14[mW] \times 168[h] = 2.35[Wh]$

$E_{R2} = P_{R2} \times t = 11.5[mW] \times 168[h] = 1.93[Wh]$

$E_{R3} = P_{R3} \times t = 4.5[mW] \times 168[h] = 0.76[Wh]$

$E_{D1} = P_{D1} \times t = 19[mW] \times 168[h] = 3.19[Wh]$

$E_{D2} = P_{D2} \times t = 21.5[mW] \times 168[h] = 3.61[Wh]$

$E_{D3} = P_{D3} \times t = 28.5[mW] \times 168[h] = 4.79[Wh]$

Real energy emitted at resistors and diodes at time 1 week = 168 hours.

$E_{R1} = P_{R1} \times t = 7.01[mW] \times 168[h] = 1.18[Wh]$

$E_{R2} = P_{R2} \times t = 7.08[mW] \times 168[h] = 1.19[Wh]$

$E_{R3} = P_{R3} \times t = 8.85[mW] \times 168[h] = 1.49[Wh]$

$E_{D1} = P_{D1} \times t = 10.49[mW] \times 168[h] = 1.76[Wh]$

$E_{D2} = P_{D2} \times t = 12.08[mW] \times 168[h] = 2.03[Wh]$

$E_{D3} = P_{D3} \times t = 24.74[mW] \times 168[h] = 4.16[Wh]$

Answers:

- Does the maximum power rating of a single resistor was exceeded ($P_{max} \leq 250$ [mA])?
  The maximum power rating of a single resistor was not exceeded. Calculations were made in step 6.
- Can the variation in tolerance of the selected real resistors (R1, R2, R3) damage the circuit in Figure 1?
  Tolerance of the real resistors are below the tolerance of the E24 series and cannot damage the circuit. Calculation were made in step 4.
- What is the total power of the system in Figure 1 (what minimum power source is required for the system)?
  The total power of the circuit is the sum of all calculated output powers in step 6.
  $P_{TOTAL} = P_{R1} + P_{R2} + P_{R3} + P_{D1} + P_{D2} + P_{D3} = 7.01[mW] + 7.08[mW] + 8.85[mW] + 10.49[mW] + 12.08[mW] + 24.74[mW] = 70.25[mW]$
- What is the total energy of the system in Figure 1 (what will be the current consumption at time t)?
  $E_{TOTAL} = P_{TOTAL} \times t = 70.25[mW] \times 168[h] = 11.802[Wh]$
  If the system will operate for 1 week then it will consume 11.8[Wh]

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